GRADO EN INTERNATIONAL BUSINESS

ESTADÍSTICA (35887)

CURSO ACADÉMICO 2018/19

Vniversitat de València
OpenCourseWare

TEMA 6
"TEOREMAS DE CONVERGENCIA Y DISTRIBUCIONES DERIVADAS DE LA NORMAL"

- **6.1. Teorema Central del Límite (consecuencias).**
- **6.2. Distribuciones derivadas de la Normal:**
  - **6.2.1. Distribución χ^2 de Pearson.**
  - **6.2.2. Distribución t de Student.**
  - **6.2.3. Distribución F de Snedecor.**

6.1. – TEOREMA CENTRAL DEL LÍMITE (CONSECUENCIAS)

1ª Consecuencia

- $X_1, X_2, \ldots\ldots X_n$ *independientes*
- $X_i \sim D(\mu, \sigma) \quad \forall i \in \{1, 2, \ldots\ldots n\}$ ***con*** $\sigma \neq 0$
- ***Si se define*** $Y_n = \sum_{i=1}^{n} X_i$

Puede aproximarse la distribución de Y_n cuando n es suficientemente grande a la de una Normal con media $n\mu$ y desviación típica $\sigma\sqrt{n}$

$(Y_n \sim N(n\mu, \sigma\sqrt{n}))$

2ª Consecuencia

- $X_1, X_2, \ldots\ldots X_n$ *independientes*
- $X_i \sim D(\mu, \sigma) \quad \forall i \in \{1, 2, \ldots\ldots n\}$ ***con*** $\sigma \neq 0$
- ***Si se define*** $Y_n = \sum_{i=1}^{n} \frac{X_i}{n}$

Puede aproximarse la distribución de Y_n cuando n es suficientemente grande a la de una Normal con media μ y desviación típica $\frac{\sigma}{\sqrt{n}}$

$(Y_n \sim N(\mu, \frac{\sigma}{\sqrt{n}}))$

ESTADÍSTICA (GIB)

6.2.- DISTRIBUCIONES DERIVADAS DE LA NORMAL

6.2.1. Distribución χ^2 de Pearson.

6.2.2. Distribución t de Student.

6.2.3. Distribución F de Snedecor.

http://go.uv.es/F4qUA75
http://pages.uv.es/piclickers/cat/MenuH_mUVies.wiki

 ESTADÍSTICA (GIB)

6.2.1.- DISTRIBUCIÓN χ^2 DE PEARSON

Definición

- $X_1, X_2, \ldots\ldots X_n$ *independientes*
- $X_i \sim N(0,1) \quad \forall i \in \{1, 2, \ldots\ldots n\}$
- *Si se define* $X = X_1^2 + X_2^2 + \ldots\ldots + X_n^2$

X sigue una distribución χ^2 de Pearson con n grados de libertad

X~ χ_n^2

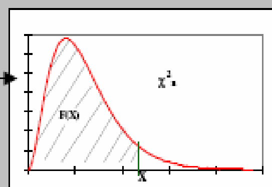
Propiedad: *la distribución χ^2 verifica el teorema de adición respecto al parámetro n*

- *X~* $\chi^2_{n_1}$
- *Y~* $\chi^2_{n_2}$
- *X e Y independientes*
- *Z=X+Y*

Z~ $\chi^2_{n_1+n_2}$

6.2.1.- DISTRIBUCIÓN χ^2 DE PEARSON

DISTRIBUCIÓN χ^2



F(X)	.005	.01	.025	.05	.10	.20	.25	.30	.40	.50	.60	.70	.75	.80	.90	.95	.975	.99	.995
n								X											
1	.0000	.0002	.0010	.0039	.0158	.0642	.1015	.1484	.2750	.4549	.7083	1.074	1.323	1.642	2.706	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.1026	.2107	.4463	.5754	.7133	1.022	1.386	1.833	2.408	2.773	3.219	4.605	5.991	7.378	9.210	10.60
3	.0717	.1148	.2158	.3518	.5844	1.005	1.213	1.424	1.869	2.366	2.946	3.665	4.108	4.642	6.251	7.815	9.348	11.34	12.84
4	.2070	.2971	.4844	.7107	1.064	1.649	1.923	2.195	2.753	3.357	4.045	4.878	5.385	5.989	7.779	9.488	11.14	13.28	14.86
5	.4117	.5543	.8312	1.145	1.610	2.343	2.675	3.000	3.655	4.351	5.132	6.064	6.626	7.289	9.236	11.07	12.83	15.09	16.75
6	.6757	.8721	1.237	1.635	2.204	3.070	3.455	3.828	4.570	5.348	6.211	7.231	7.841	8.558	10.64	12.59	14.45	16.81	18.55
7	.9893	1.239	1.690	2.167	2.833	3.822	4.255	4.671	5.493	6.346	7.283	8.383	9.037	9.803	12.02	14.07	16.01	18.48	20.28
8	1.344	1.647	2.180	2.732	3.490	4.594	5.071	5.527	6.423	7.344	8.351	9.524	10.22	11.03	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	5.380	5.899	6.393	7.357	8.343	9.414	10.66	11.39	12.24	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	6.179	6.737	7.267	8.295	9.342	10.47	11.78	12.55	13.44	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	6.989	7.584	8.148	9.237	10.34	11.53	12.90	13.70	14.63	17.27	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	7.807	8.438	9.034	10.18	11.34	12.58	14.01	14.85	15.81	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	8.634	9.299	9.926	11.13	12.34	13.64	15.12	15.98	16.98	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	9.467	10.17	10.82	12.08	13.34	14.69	16.22	17.12	18.15	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	10.31	11.04	11.72	13.03	14.34	15.73	17.32	18.25	19.31	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	11.15	11.91	12.62	13.98	15.34	16.78	18.42	19.37	20.47	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	12.00	12.79	13.53	14.94	16.34	17.82	19.51	20.49	21.61	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	12.86	13.68	14.43	15.89	17.34	18.87	20.60	21.60	22.76	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	13.72	14.56	15.35	16.85	18.34	19.91	21.69	22.72	23.90	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	14.58	15.45	16.27	17.81	19.34	20.95	22.77	23.83	25.04	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	15.44	16.34	17.18	18.77	20.34	21.99	23.86	24.93	26.17	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	16.31	17.24	18.10	19.73	21.34	23.03	24.94	26.04	27.30	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	17.19	18.14	19.02	20.69	22.34	24.07	26.02	27.14	28.43	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	18.06	19.04	19.94	21.65	23.34	25.11	27.10	28.24	29.55	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	18.94	19.94	20.87	22.62	24.34	26.14	28.17	29.34	30.68	34.38	37.65	40.65	44.31	46.93
30	13.79	14.95	16.79	18.49	20.60	23.36	24.48	25.51	27.44	29.34	31.32	33.53	34.80	36.25	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	32.34	33.66	34.87	36.16	39.34	41.62	44.16	45.62	47.27	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	41.45	42.94	44.31	46.86	49.33	51.89	54.72	56.33	58.16	63.17	67.51	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	50.64	52.29	53.81	56.62	59.33	62.13	65.23	66.98	68.97	74.40	79.08	83.30	88.38	91.95
70	43.27	45.44	48.76	51.74	55.33	59.90	61.70	63.35	66.40	69.33	72.36	75.69	77.58	79.71	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	69.21	71.14	72.92	76.19	79.33	82.57	86.12	88.13	90.41	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	78.56	80.62	82.51	85.99	89.33	92.76	96.52	98.65	101.1	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.86	87.95	90.13	92.13	95.81	99.33	102.9	106.9	109.1	111.7	118.5	124.3	129.6	135.8	140.2

n > 30 $\quad \chi^2_\alpha = \frac{1}{2}\left(\lambda_\alpha + \sqrt{2n-1}\right)^2 \quad \lambda_\alpha$ de la N(0,1)

- Gráfica de la función de densidad: 1
- Valores grados de libertad: 2
- Aproximación por la distribución Normal: 3
- Valores función de distribución: 4
- Valores de la variable: 5

6.2.2.- DISTRIBUCIÓN t DE STUDENT

Definición

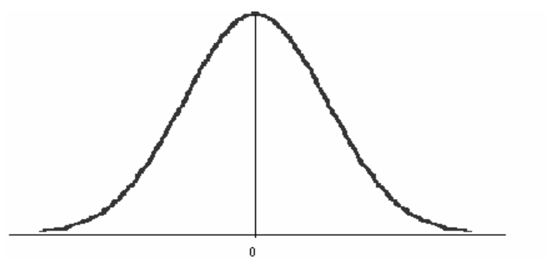
- ***X ~ N(0, 1), Y ~*** χ^2_n
- ***X e Y independientes***
- ***Si se define*** $t = \dfrac{X}{\sqrt{\dfrac{Y}{n}}}$

t sigue una distribución t de Student con n grados de libertad

$$t \sim t_n$$

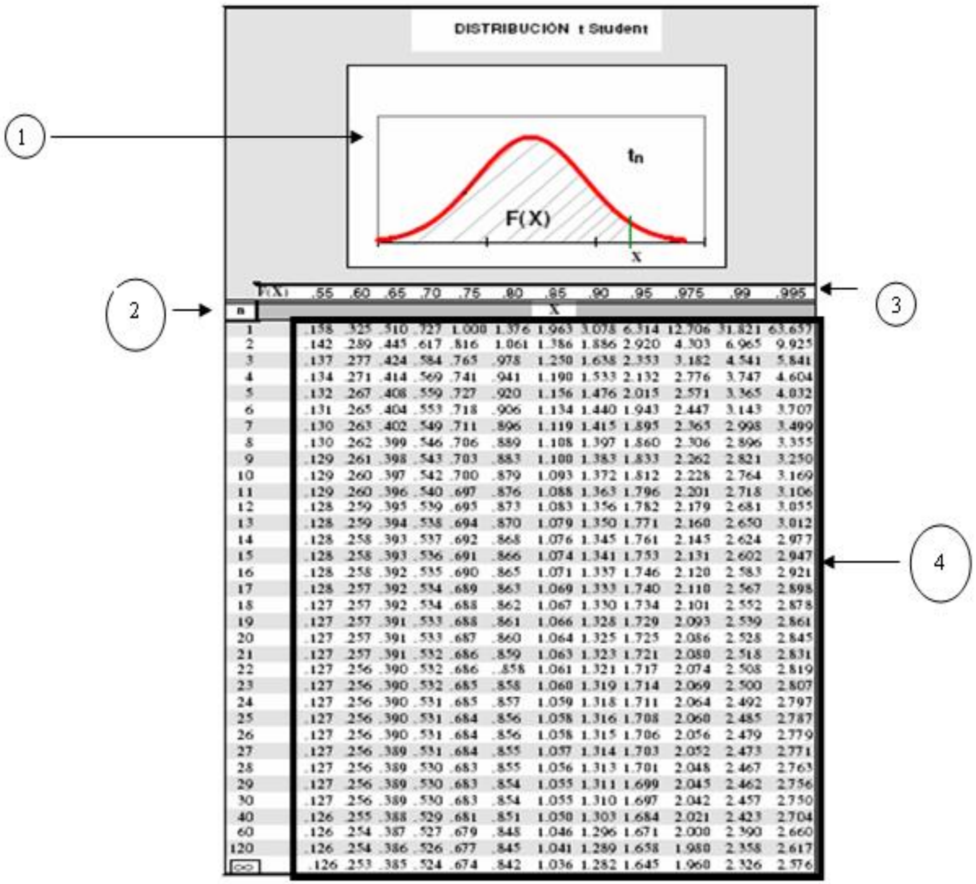
Propiedades:

1ª μ=Me=Mo=0



2ª **Para n suficientemente grande puede aproximarse la distribución de t a la de una N(0, 1)**

6.2.2.- DISTRIBUCIÓN t DE STUDENT



F(X)	.55	.60	.65	.70	.75	.80	.85	.90	.95	.975	.99	.995
n	X											
1	.158	.325	.510	.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845
21	.127	.257	.391	.532	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831
22	.127	.256	.390	.532	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819
23	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807
24	.127	.256	.390	.531	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797
25	.127	.256	.390	.531	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787
26	.127	.256	.390	.531	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779
27	.127	.256	.389	.531	.684	.855	1.057	1.314	1.703	2.052	2.473	2.771
28	.127	.256	.389	.530	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763
29	.127	.256	.389	.530	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756
30	.127	.256	.389	.530	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750
40	.126	.255	.388	.529	.681	.851	1.050	1.303	1.684	2.021	2.423	2.704
60	.126	.254	.387	.527	.679	.848	1.046	1.296	1.671	2.000	2.390	2.660
120	.126	.254	.386	.526	.677	.845	1.041	1.289	1.658	1.980	2.358	2.617
∞	.126	.253	.385	.524	.674	.842	1.036	1.282	1.645	1.960	2.326	2.576

- Gráfica de la función de densidad: 1
- Valores grados de libertad: 2
- Valores función de distribución: 3
- Valores de la variable: 4

ESTADÍSTICA (GIB)

6.2.3.- DISTRIBUCIÓN F DE SNEDECOR

Definición

- ***X ~*** χ^2_m ***, Y ~*** χ^2_n
- ***X e Y independientes***
- ***Si se define*** $F = \dfrac{X/m}{Y/n} = \dfrac{n}{m} \cdot \dfrac{X}{Y}$

F sigue una distribución F de Snedecor con m grados de libertad en el numerador y n grados de libertad en el denominador

F ~ $F_{m,\ n}$

Propiedad de inversión de la F de Snedecor:

Si **F ~ $F_{m,\ n}$**

$\left(\dfrac{1}{F}\right)$ **~ $F_{n,\ m}$**