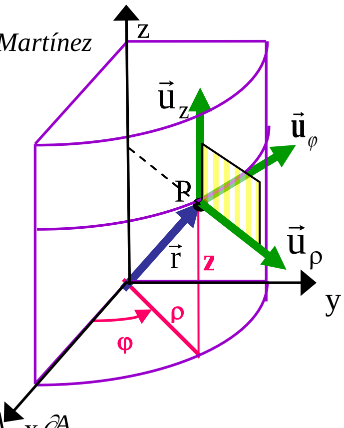


COORDENADES CILÍNDRIQUES

$\rho = \sqrt{x^2+y^2}$ $\qquad \mathbf{x} = \rho\cos\varphi \qquad \mathbf{d\vec{r}} = d\rho\,\vec{u}_\rho + \rho\, d\varphi\,\vec{u}_\varphi + dz\,\vec{u}_z$

$\varphi = tg^{-1}\dfrac{y}{x}$ $\qquad \mathbf{y} = \rho\sin\varphi \qquad \mathbf{d\vec{S}} = \rho\, d\varphi\, dz\,\vec{u}_\rho + d\rho\, dz\,\vec{u}_\varphi + \rho\, d\varphi\, d\rho\,\vec{u}_z$

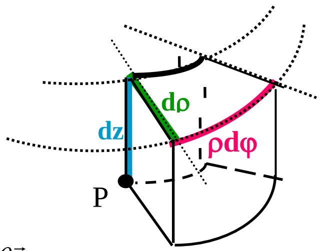
$\mathbf{z} = z$ $\qquad \mathbf{z} = z \qquad \mathbf{dV} = \rho\, d\rho\, d\varphi\, dz$

GRADIENT $\quad \vec{\nabla}\psi = \dfrac{\partial\psi}{\partial\rho}\vec{u}_\rho + \dfrac{1}{\rho}\dfrac{\partial\psi}{\partial\varphi}\vec{u}_\varphi + \dfrac{\partial\psi}{\partial z}\vec{u}_z$

DIVERGÈNCIA $\quad \vec{\nabla}\cdot\vec{A} = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}(\rho A_\rho) + \dfrac{1}{\rho}\dfrac{\partial A_\varphi}{\partial\varphi} + \dfrac{\partial A_z}{\partial z}$

ROTACIONAL $\quad \vec{\nabla}\times\vec{A} = \left(\dfrac{1}{\rho}\dfrac{\partial A_z}{\partial\varphi} - \dfrac{\partial A_\varphi}{\partial z}\right)\vec{u}_\rho + \left(\dfrac{\partial A_\rho}{\partial z} - \dfrac{\partial A_z}{\partial\rho}\right)\vec{u}_\varphi + \dfrac{1}{\rho}\left(\dfrac{\partial}{\partial\rho}(\rho A_\phi) - \dfrac{\partial A_\rho}{\partial\varphi}\right)\vec{u}_z$

LAPLACIANA $\quad \Delta\psi = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}\left(\rho\dfrac{\partial\psi}{\partial\rho}\right) + \dfrac{1}{\rho^2}\dfrac{\partial^2\psi}{\partial\varphi^2} + \dfrac{\partial^2\psi}{\partial z^2}$



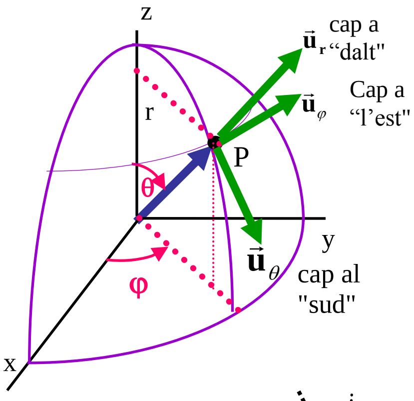
COORDENADES ESFÉRIQUES

$\mathbf{r} = \sqrt{x^2+y^2+z^2}$

$\varphi = tg^{-1}\dfrac{y}{x}$ $\qquad \mathbf{x} = r\sin\theta\cos\varphi \qquad \mathbf{d\vec{r}} = dr\,\vec{u}_r + r\,\mathbf{sin}\theta\, d\varphi\,\vec{u}_\varphi + r\, d\theta\,\vec{u}_\theta$

$\theta = \mathbf{cos}^{-1}\dfrac{z}{r}$ $\qquad \mathbf{y} = r\sin\theta\sin\varphi \qquad \mathbf{d\vec{S}} = r^2\,\mathbf{sin}\theta d\theta\, d\varphi\,\vec{u}_r + r\, dr\, d\theta\,\vec{u}_\varphi + r\,\mathbf{sin}\theta dr\, d\varphi\,\vec{u}_\theta$

$\mathbf{z} = r\cos\theta \qquad \mathbf{dV} = r^2\,\mathbf{sin}\theta dr\, d\theta\, d\varphi$



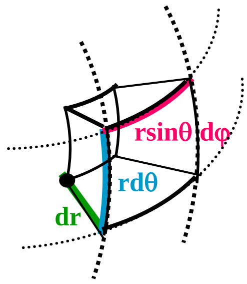
GRADIENT $\quad \vec{\nabla}\psi = \dfrac{\partial\psi}{\partial r}\vec{u}_r + \dfrac{1}{r\sin\theta}\dfrac{\partial\psi}{\partial\varphi}\vec{u}_\varphi + \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}\vec{u}_\theta$

DIVERGÈNCIA $\quad \vec{\nabla}\cdot\vec{A} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2 A_r) + \dfrac{1}{r\sin\theta}\dfrac{\partial A_\varphi}{\partial\varphi} + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\theta}(A_\theta\sin\theta)$

ROTACIONAL $\quad \vec{\nabla}\times\vec{A} = \dfrac{1}{r\sin\theta}\left(\dfrac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \dfrac{\partial A_\theta}{\partial\varphi}\right)\vec{u}_r +$

$\dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(rA_\theta) - \dfrac{\partial A_r}{\partial\theta}\right)\vec{u}_\varphi + \dfrac{1}{r\sin\theta}\left(\dfrac{\partial A_r}{\partial\varphi} - \sin\theta\dfrac{\partial}{\partial r}(rA_\varphi)\right)\vec{u}_\theta$

LAPLACIÀ $\quad \Delta\psi = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\psi}{\partial r}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2\psi}{\partial\varphi^2} + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial\psi}{\partial\theta}\right)$



RELACIONS VECTORIALS

$\vec{\nabla}(\psi\phi) = \psi\,\vec{\nabla}\phi + (\vec{\nabla}\psi)\,\phi$

$\vec{\nabla}(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\vec{\nabla})\vec{b} + (\vec{b}\cdot\vec{\nabla})\vec{a} + \vec{a}\times(\vec{\nabla}\times\vec{b}) + \vec{b}\times(\vec{\nabla}\times\vec{a})$

$\vec{\nabla}\cdot(\psi\vec{a}) = \vec{a}\cdot\vec{\nabla}\psi + \psi\vec{\nabla}\cdot\vec{a}$

$\vec{\nabla}\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\vec{\nabla}\times\vec{a}) - \vec{a}\cdot(\vec{\nabla}\times\vec{b})$

$\vec{\nabla}\times(\psi\,\vec{a}) = \vec{\nabla}\psi\times\vec{a} + \psi\,\vec{\nabla}\times\vec{a}$

$\vec{\nabla}\times\vec{\nabla}\psi = 0$

$\vec{\nabla}\times(\vec{a}\times\vec{b}) = \vec{a}(\vec{\nabla}\cdot\vec{b}) - \vec{b}(\vec{\nabla}\cdot\vec{a}) + (\vec{b}\cdot\vec{\nabla})\vec{a} - (\vec{a}\cdot\vec{\nabla})\vec{b}$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - \vec{\nabla}^2\vec{a}$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{a}) = 0$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$

$\vec{\nabla}\left(\dfrac{\vec{R}}{R^3}\right) = \vec{\nabla}\left(\dfrac{\vec{u}_R}{R^2}\right) = 4\pi\delta(\vec{R})$

$\vec{\nabla}\left(\dfrac{1}{R}\right) = -\dfrac{\vec{R}}{R^3} = -\dfrac{\vec{u}_R}{R^2}$

$\vec{\nabla}\left[\vec{\nabla}\left(\dfrac{1}{R}\right)\right] = \vec{\nabla}\left[-\dfrac{\vec{R}}{R^3}\right] = -4\pi\delta(\vec{R})$

$\int_V \vec{\nabla}\cdot\vec{A}\; dV = \oint_{S(V)} \vec{A}\cdot d\vec{S} \qquad \int_S \vec{\nabla}\times\vec{A}\cdot d\vec{S} = \oint_{L(S)} \vec{A}\cdot d\vec{l}$

$\int_V \vec{\nabla}\times\vec{A}\; dV = -\oint_{S(V)} \vec{A}\times d\vec{S} \qquad \int_S d\vec{S}\times\nabla\psi = \oint_{L(S)} \psi\, d\vec{l}$

INTEGRALS MÉS HABITUALS

(*CRC Standard Mathematical Tables*)

43. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

157. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left(x+\sqrt{x^2 \pm a^2}\right)$

158. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{|a|} sec^{-1}\frac{x}{a}$

159. $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\log\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$

162. $\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$

165. $\int \frac{dx}{\left(x^2 \pm a^2\right)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$

166. $\int \frac{x\,dx}{\left(x^2 \pm a^2\right)^{3/2}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$

182. $\int \frac{x^2\,dx}{\left(x^2 \pm a^2\right)^{3/2}} = \frac{-x}{\sqrt{x^2 \pm a^2}} + \log\left(x+\sqrt{x^2 \pm a^2}\right)$

183. $\int \frac{x^3\,dx}{\left(x^2 \pm a^2\right)^{3/2}} = \sqrt{x^2 \pm a^2} \pm \frac{a^2}{\sqrt{x^2 \pm a^2}}$

184. $\int \frac{dx}{x\left(x^2+a^2\right)^{3/2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\log\frac{a+\sqrt{x^2+a^2}}{x}$

185. $\int \frac{dx}{x\left(x^2-a^2\right)^{3/2}} = -\frac{1}{a^2\sqrt{x^2 \pm a^2}} - \frac{1}{|a^3|} sec^{-1}\frac{x}{a}$

186. $\int \frac{dx}{x^2\left(x^2 \pm a^2\right)^{3/2}} = -\frac{1}{a^4}\left(\frac{\sqrt{x^2 \pm a^2}}{x} + \frac{x}{\sqrt{x^2 \pm a^2}}\right)$

CONDICIONS D'ORTOGONALITAT

$$\int_0^{2\pi} \mathbf{cos}(m\varphi)\,\mathbf{cos}(n\varphi)\,d\varphi = \pi\delta_{m,n}$$

$$\int_0^{a} \mathbf{cos}(k_m x)\,\mathbf{cos}(k_n x)\,dx = \frac{a}{\mathbf{2}}\delta_{m,n}$$

$$\int_0^{2\pi} \mathbf{sin}(m\varphi)\,\mathbf{sin}(n\varphi)\,dx = \pi\delta_{m,n}$$

$$\int_0^{a} \mathbf{sin}(k_m x)\,\mathbf{sin}(k_n x)\,dx = \frac{a}{\mathbf{2}}\delta_{m,n}$$

$$\int_0^{2\pi} \mathbf{sin}(m\varphi)\,\mathbf{cos}(n\varphi)\,d\varphi = \mathbf{0}$$

$$\int_0^{a} \mathbf{sin}(k_m x)\,\mathbf{cos}(k_m x)\,dx = \mathbf{0}$$

$$\int_0^{\pi} P_m(\mathbf{cos}\theta)P_n(\mathbf{cos}\theta)\,\mathbf{sin}\theta\,d\theta = \frac{\mathbf{2}}{\mathbf{2n+1}}\delta_{m,n}$$

(m=1,2,3… ; n=1,2,3…)

POLINOMIS DE LEGENDRE

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{2}(3\mathbf{cos}^2\theta - 1)$$

DESENVOLUPAMENTS EN SÈRIE MÉS UTILITZATS:

$$(1\pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$$

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^{4\ldots}$$

$$\log\frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5}\ldots\right)$$