

# Sampling from a (normal) population

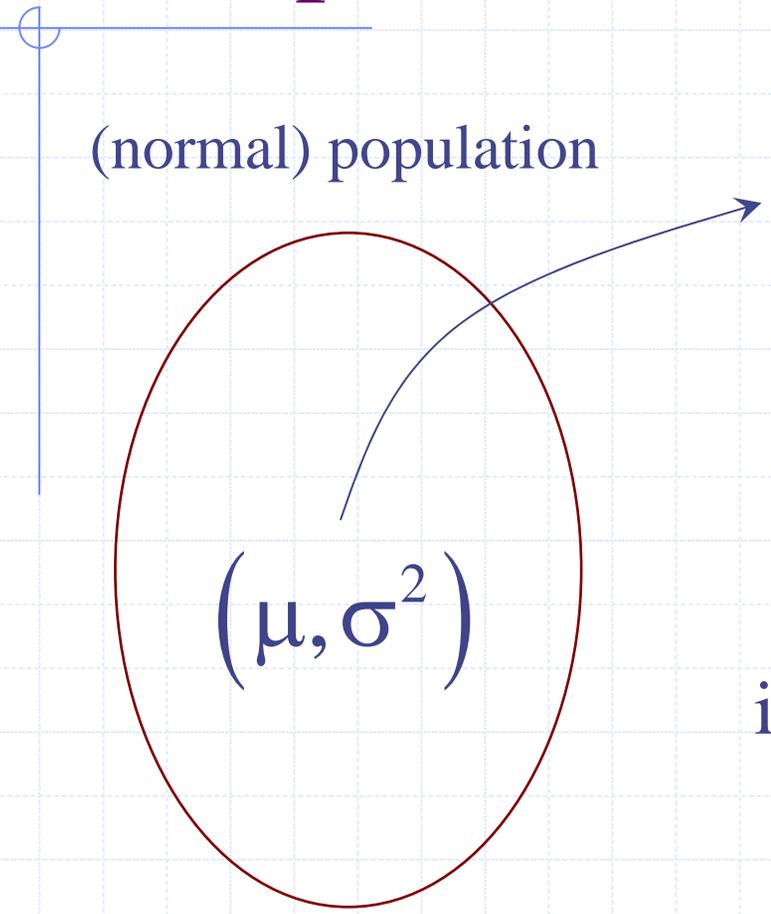
# Basics

- ◆ I assume you are familiar with the basic problem of estimating the mean from a (normal) population, given a random sample.
- ◆ Imagine you have a random sample from a (normal) population with mean  $\mu$  and variance  $\sigma^2$ .

# Set up

(normal) population

$$\{x_i\}_{i=1}^n$$


$$(\mu, \sigma^2)$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

is used as an estimator for  $\mu$

# Properties of estimators

- ◆ What are the properties of the sample mean,  $\bar{x}$ , as an estimator of the population mean,  $\mu$ ?
- ◆ Please, note that estimators are random variables (they depend on the random sample!).

# Properties of estimators

1. The sample mean is an unbiased estimator of the population mean,  $\mu$ .

$$E(\bar{x}) = \mu$$

2. The precision of the estimator, as given by his variance, is:

$$Var(\bar{x}) = \frac{\sigma^2}{n}$$

# Properties of estimators

3. If the population is normal, then

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

A result than can be used to perform hypothesis testing on  $\mu$ .

# Properties of estimators

4. Otherwise, we can get an approximation, based on the Central Limit Theorem,

$$\sqrt{n} \frac{(\bar{x} - \mu)}{\sigma} \xrightarrow{d} N(0,1)$$

Hence,

$$\bar{x} \stackrel{aprox}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

# Hypothesis testing

- ◆ Note that, unless  $\sigma^2$  is known this additional population parameter will have to be estimated before we can use the above result for hypothesis testing.
- ◆ What we are going to see in the course is basically an extension of this basic set up.