Sampling from a (normal) population

Basics

- I assume you are familiar with the basic problem of estimating the mean from a (normal) population, given a random sample.
- Imagine you have a random sample from a (normal) population with mean μ and variance σ^2.

Set up

(normal) population

$$\left(\mu, \sigma^2\right)$$

$$\left\{x_i\right\}_{i=1}^{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

is used as an estimator for μ

Properties of estimators

- What are the properties of the sample mean, $\bar{x}$, as an estimator of the population mean, μ?

- Please, note that estimators are random variables (they depend on the random sample!).

Properties of estimators

1. The sample mean is un unbiased estimator of the population mean, μ.

$$E(\overline{x}) = \mu$$

2. The precision of the estimator, as given by his variance, is:

$$Var(\overline{x}) = \frac{\sigma^2}{n}$$

Properties of estimators

3. If the population is normal, then

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

A result than can be used to perform hypothesis testing on μ.

Properties of estimators

4. Otherwise, we can get an approximation, based on the Central Limit Theorem,

$$\sqrt{n}\frac{\left(\overline{x}-\mu\right)}{\sigma} \xrightarrow{d} N(0,1)$$

Hence,

$$\overline{x} \overset{aprox}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

Hypothesis testing

- Note that, unless σ^2 is known this additional population parameter will have to be estimated before we can used the above result for hypothesis testing.

- What we are going to see in the course is basically an extension of this basic set up.